# 1 Introduction

For most of the length of rivers in the world, the slope (the rate of decline in elevation over space) of the river follows closely with the slope of the surrounding landscape: rivers in mountains are steep and rivers near the ocean are shallow. The water-surface in these regions is said to be "slope mediated" (Figure 1). Consider the cross-section of a river: there are two banks on either side of the channel (called *levees*), and the channel is deepest somewhere near the middle. So, when a flood passes through this river cross-section, bringing more water into the area, the river intuitively becomes deeper and the elevation of the water surface is raised. This increase in flow depth can overcome the channel levees and cause the river to flow out and across the landscape—this is effectively what "flooding" is, and it has obvious downsides for a society living near the river.

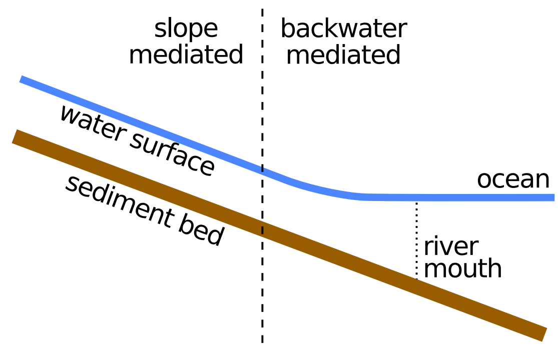



Figure 1: long-profile of a river entering into an ocean.

Now consider a river very near the end of its length, where it enters into an ocean. The volume of water leaving the river is very small with respect to the amount of water in the receiving ocean. As a result, no matter how much water is pumped out from the river into the ocean, the water surface elevation in the ocean remains fixed; so flooding does not raise the elevation of the water everywhere. Furthermore, the slope of the water surface in the ocean is effectively flat. This means that there must also be a transition from the upstream slope mediated water surface to a slope of effectively zero—this region is called the *backwater region*, and the water surface slope here is said to be "backwater mediated" (Figure 1).

A helpful example is to consider the Mississippi River, in Louisiana. This river is one of the largest rivers in the world, discharging more than 30,000 m$^3$/s of water into the Gulf of Mexico during flood. The Gulf of Mexico has a volume of approximately 2.5 million km$^3$ of water. If we generously say that the Mississippi River is flooding for two months out of the year, that means that the river discharges

$$30,000 \text{ m}^3\text{s}^{-1} \cdot 86,400 \text{ s d}^{-1} \cdot 60 \text{ d} \cdot 10^{-9} \text{ km}^3\text{m}^{-3} = 155.5 \text{ km}^3 \text{ water} \quad (1)$$

which is less than 1 hundred-thousandth ($< 10^{-6}$) of the water volume of the Gulf of Mexico. This quick calculation makes it easy to see how impossible it is for a river to change the volume of water and elevation or slope of the ocean it delivers water to.

Interestingly, this fixed elevation of the ocean actually affects the river far upstream of the mouth. Recall that the flow depth of a river increases during flood—however, this cannot be the case at the downstream end of the river, because the elevation here is known to be fixed. Therefore, not only does the water-surface slope transition through the backwater region, but the magnitude of the variation of the water surface elevation during flood is also diminished through the backwater region.

Of course, high variation in the elevation of the river allows for the possibility of overflowing the confinement of the channel and flooding out over the land the surrounds the river, where many millions of people reside. The following sections of this module explore a model for the Mississippi River near the outlet to the Gulf of Mexico, to determine how the backwater affects flooding in this area.

## 2 Idealized model

Using two physical laws (the conservation of mass and momentum) a prediction of the flow depth can be made for a given water discharge (e.g., Chow (1959)). This allows for testing of the statements made above:

1. the slope of the river is reduced downstream
2. stage variability during flood is diminished downstream

Open the module downloaded with this worksheet, and familiarize yourself with the interface. The large plot at the top of the interface displays the *long-profile* of the model of the Mississippi River; a long-profile is a slice through the river centerline from upstream to downstream. The Mississippi River (Figure 2) is a large river by measure of water discharge, and has a very low sloping channel bed.

The model is a simplified representation of the real river system, however, and a few assumptions made to produce the model are that 1) the width of the river is constant, 2) the river is shaped like a rectangle in cross-section, 3) the river bed has a constant slope, and 4) the amount of water in the river is constant along the entire length.

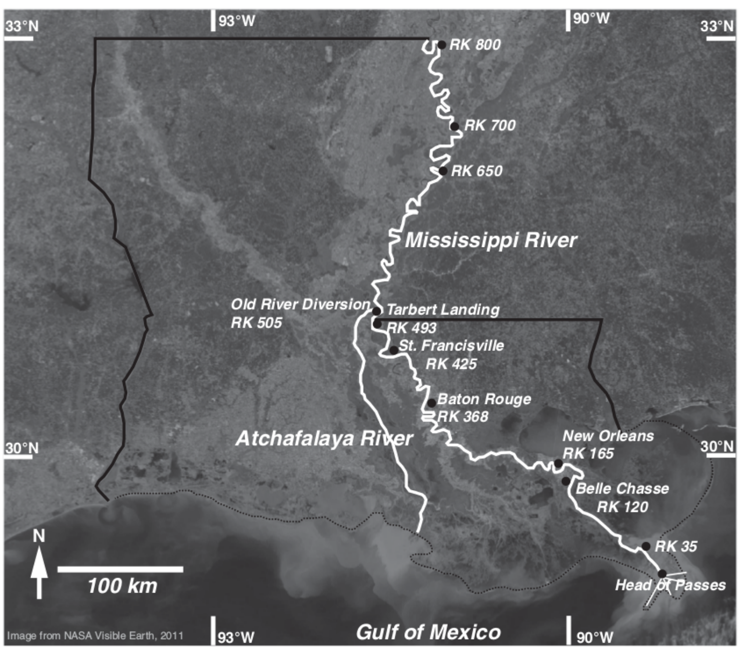



Figure 2: the lower 800 river kilometers of the Mississippi River; the state of Louisiana is outlined. Locations labeled are referenced in this exercise. Figure from Nittrouer et al. (2012).

**Select one of these assumptions and describe in a sentence why it may be invalid:**

---

---

For now, proceed under the assumption that the model is an accurate characterization of the actual Mississippi River—this will be returned to later, though. In order to test the first statement (that the slope of the river is reduced downstream) we can pretty simply just inspect the plot in the interface. First, reset the discharge to 10,000 $m^3/s$ (use reset button). **What is the upstream extent of the backwater? I.e., what river kilometer (RK) does the water surface slope begin to change?** _________ **What river kilometer does the slope no longer change any further downstream?** _________

The length of channel between the values identified above is the backwater region for a discharge of 10,000 $m^3/s$. The length, and location of transition, may change for different discharges. **How does the length of the backwater region change for different discharges? Select another discharge and explain:**

---

---

---

---

---

**Does the backwater always initiate at the same location upstream for all discharges? What happens to the backwater region when the discharge is increased? You may wish to graph the model response to different discharges to answer this question.**

---

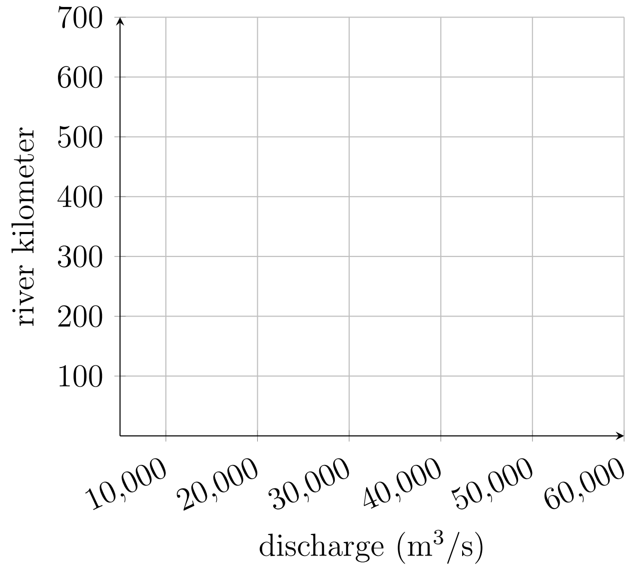
---

---

---

---

---




The measurements you've made so far describe the change in slope through the backwater (i.e., statement 1), however an assessment of the variation in water surface elevation is necessary to evaluate statement 2. The elevation of the water surface measured in a river is called the *stage*. The stage variation is the result of a change in flow depth; both are tabulated at a few locations along the course of the Mississippi River (Figure 2) and shown in the table at the bottom of the interface. Now, reset the discharge to 10,000 $m^3/s$ (use reset button).

**What is the flow depth and stage at Head of Passes for a 10,000 $m^3/s$ discharge?** _______ _______ Head of Passes is the mouth of the river, where the river water enters the Gulf of Mexico and is no longer contained within the channel. It was stated that it is impossible to raise the elevation of the Gulf of Mexico through increasing river discharge, but that the stage of the river increases upstream, nonetheless. **What is the flow depth and stage at Head of Passes for a ∼40,000 $m^3/s$ discharge?** _______ _______ **Is there much change in the flow depth or stage at Head of Passes?**

---

Repeat these measurements for two more of the locations listed in the table. **What is the stage change for these locations? Complete the following table:**

| Location name: | Head of Passes | | |
|---|---|---|---|
| 10,000 $m^3/s$ stage (m) | | | |
| 40,000 $m^3/s$ stage (m) | | | |
| stage change (m) | | | |

**Explain why the variation in stage you measured at each of the locations fits or does not fit with the framework for the backwater region described earlier:**

---

---

---

There is an index in the table that tracks the inundation of a hypothetical levee system that is developed to contain a regular flooding discharge of the Mississippi River: 35,000 $m^3/s$. It is important to point out at this time that the model used here does not provide an accurate depiction of how much flooding will occur for different discharges (i.e., how much flow goes over the banks of the river), but rather provides only an estimation of when the banks of the river will become overflowed; this is because of an assumption baked into the model and beyond the scope of this module.

Vary the discharge and explore the conditions that lead to the overflowing of the levees.
**What is the minimum discharge to overflow the levee?** _______
**Which location is the first to become inundated?** ____________

**Why is this the case? Explain in the context of the backwater flow properties we've discussed above.**

**Where should the river levees be fortified (built taller) in order to prevent the river from causing flooding? Why?**

## 3 Comparing with real data

The predictions made above are all calculated based on an idealized model for the Mississippi River that uses a channel bed with constant slope. Data were compiled by Nittrouer et al. (2012) and used to calibrate a model similar to the one used in this module. Check the box labeled "show thalweg". The *thalweg* is the name for the deepest part of the river—by checking this box, the actual measured elevation of the Mississippi River thalweg is shown. **Do you expect the model results to differ if the actual data were used instead of the idealized constant-slope channel bed? Explain:**

Now check the box labeled "show water lines". This option shows measured stages of the river for various discharges. **Compare the model predictions for the same discharge to each of the measured water lines. What causes the model predictions to break down? Where do the model predictions seem to fail?**

## 4 Navigability in the channel

Imagine there are no bridges over the Mississippi River to block the path of ships that wish to navigate upstream. You are the captain of a relatively large cargo ship that has a *draft* (draft is how far a ship's bottom extends below the water surface) of 10.2 m when fully loaded with goods. You are told to dock as far up the river as possible to minimize the cost of trucking the goods farther inland. **Without knowing the water depth at the time you sail up the river, what is the farthest location along the river you would safely navigate to? Estimate to an approximate river kilometer. Why would it not be wise to travel any further?**

## References

V. T. Chow. *Open Channel Hydaulics*. McGraw-Hill civil engineering series. 1959.

J. A. Nittrouer, J. Shaw, M. P. Lamb, and D. Mohrig. Spatial and temporal trends for water-flow velocity and bed-material sediment transport in the lower Mississippi River. *Geological Society of America Bulletin*, 124(3-4):400–414, Mar. 2012. ISSN 0016-7606, 1943-2674. doi: 10.1130/B30497.1.

This module utilizes a Matlab Compiled Runtime (MCR) program for backend calculations and plotting of the model results. The program accompanying this module is to be run as a standalone application, but relies on the Matlab MCR; the program can be installed with a download at `http://www.coastalsustainability.rice.edu/outreach/`. For help troubleshooting the Matlab executable, please visit `https://www.mathworks.com/products/compiler/mcr.html`. Model parameterization follows Nittrouer et al. (2012); the parameters can be viewed in the source code at `https://github.com/amoodie/research_outreach/flooding_risk`.

The module was created by Andrew J. Moodie as part of an National Science Foundation funded research project assessing the sustainability of anthropogenically influenced deltas. The research is supported by Grant No. 1427262 and an NSF Graduate Research Fellowship to A.J.M. under Grant No. 1450681. Any opinion, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. For more information, please visit `http://www.coastalsustainability.rice.edu/outreach/`.